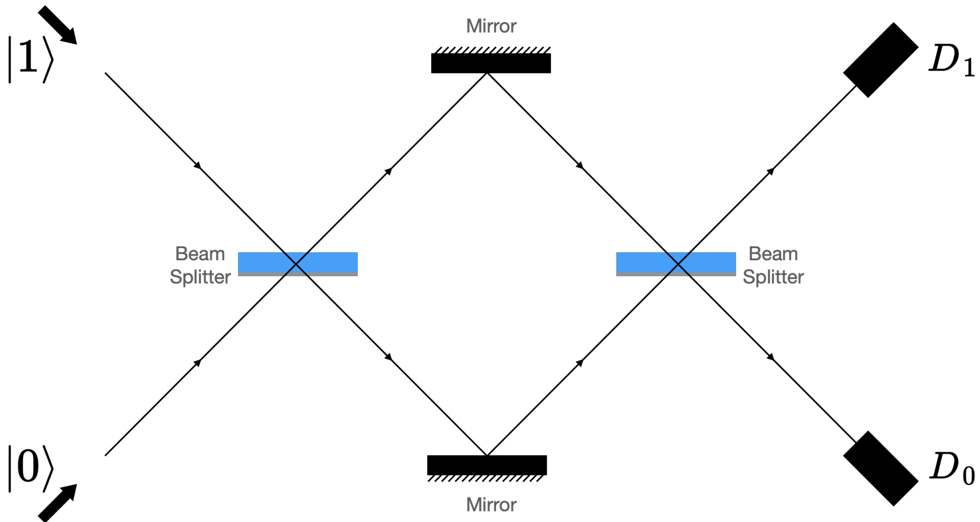
Single Photon Quantum Interference In A Mach-Zehnder Interferometer



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Arrow Entanglement - Logic Masters Deutschland Webpage

Consider the setup in the image, known as a Mach-Zehnder interferometer, which is repeated in various forms of the puzzle Arrow Entanglement. At all times the state of a photon can be written as a column vector, and the photon enters the first beam splitter in either $|1\rangle$ or $|0\rangle$ where:

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In a quantum state, each element of the vector gives the 'probability amplitude' of the photon existing in that state (the first for the upper path and the second for the lower), with each probability calculated as the square of the respective amplitudes. As the initial location of a photon is known, for initial state $|1\rangle$ the probability of finding it the upper path is $1^2 = 1$ while the probability for the lower path is $0^2 = 0$, and vice versa for $|0\rangle$.

A beam splitter is technically a half-silvered mirror, which reflects/transmits with $\frac{1}{2}$ probability each. A reflection off the silvered surface gets a phase shift of π ($e^{i\pi} = -1$), akin to inverting a sine wave, so that peaks become troughs and vice versa. This can be expressed a Hadamard matrix $\mathbf{H}$, with the position of the -1 dependent on the choice of which reflection gets the phase shift. In the image, the silvered surface affects reflections on the lower path.

$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{or} \quad \mathbf{H}_{upper} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{for silvered upper path}$$

Now consider the effect of the beam splitter on a single photon arriving in either $|1\rangle$ or $|0\rangle$:

$$|1_A\rangle = \mathbf{H}\,|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \times 1 & + & 1 \times 0 \\ 1 \times 1 & + & -1 \times 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|0_A\rangle = \mathbf{H}\,|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \times 0 & + & 1 \times 1 \\ 1 \times 0 & + & -1 \times 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The photon now exists in a quantum state, where it is simultaneously in both exit paths from the first beam splitter. Unless and until it is detected, it has to be treated as travelling along both paths in this state. This is analogous to Schrödinger's Cat, the well known thought experiment where a cat in a box is both alive and dead before the box is opened. Once the box is opened, the quantum state collapses because the state is now known after observation. Consider removing the second beam splitter, in which case the upper path hits D_0 and the lower path hits D_1 as the paths still cross. From the probability amplitudes, all probabilities are $\left(\pm\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$, so a photon that started in $|1\rangle$ or $|0\rangle$ has a $\frac{1}{2}$ of hitting either detector.

Now consider the image, where the photon hits the second beam splitter. We model this once again with a Hadamard matrix:

$$|1_B\rangle = \mathbf{H}\,|1_A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\times 1 & + & 1\times 1\\ 1\times 1 & + & -1\times 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\ 0\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$$

$$|0_B\rangle = \mathbf{H}\,|0_A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\times 1 & + & 1\times -1\\ 1\times 1 & + & -1\times -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\ 2\end{pmatrix} = \begin{pmatrix}0\\ 1\end{pmatrix}$$

Bizarrely, the result of the calculation is that all photons that start in $|1\rangle$ hit D_1, and all photons that start in $|0\rangle$ hit D_0. This is because the phase shifts from the silvered side cause the photon to destructively interfere with itself. This result can only be understood from the photon existing in both paths between the beam splitters simultaneously, which is a very counterintuitive idea. It should be noted that switching the silvered side for one of the beam splitters, and using $\mathbf{H}_{upper}$ instead of $\mathbf{H}$, switches which initial state reaches each detector.

Now consider the possibility that the photon is observed before hitting the second beam splitter. Consider a photon in initial state $|1\rangle$ with an extra detector in the lower path. Now, that detector will have a $\frac{1}{2}$ probability of being hit by the photon while it is in the state $|1_A\rangle$, but there is also a $\frac{1}{2}$ probability of the photon continuing along the upper path in the state $\frac{1}{\sqrt{2}}|1\rangle$. The photon would then enter the second beam splitter and a Hadamard operator would transform the output as:

$$\mathbf{H}\frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\times 1 & + & 1\times 0\\ 1\times 1 & + & -1\times 0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\ 1\end{pmatrix}$$

Now there is a probability of $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ of the photon being detected in either D_1 or D_0. Similar calculations will reveal this to be true for either initial state with the detector in either path.

Now consider the effect of a pass-through detector, again using initial state $|1\rangle$ and placing the detector on the lower path. Once again, the detector has a $\frac{1}{2}$ probability of being hit, but now it allows the photon to continue. However, we do not obtain $|1_B\rangle$ in this case and all photons hitting D_1. This is because the quantum state has now collapsed; we know where the photon is, so both paths into the second beam splitter must be treated independently. Following either path gives the same result as before, a $\frac{1}{4}$ chance of hitting the detector, but now the two probabilities for each detector must be summed, so both D_1 and D_0 have a $\frac{1}{2}$ chance of being hit.

This is one of many unusual results from quantum mechanics, and it has basis in concepts such as wave-particle duality. Understanding quantum mechanics in detail requires a good knowledge of maths, particularly matrix algebra, but a qualitative understanding of broad principles is not beyond most people. I encourage you to explore further and expand your horizons!