Square End Table – Full Solution Guide

This is a full solution guide to my puzzle *Square End Table*, and so spoilers are ahead.

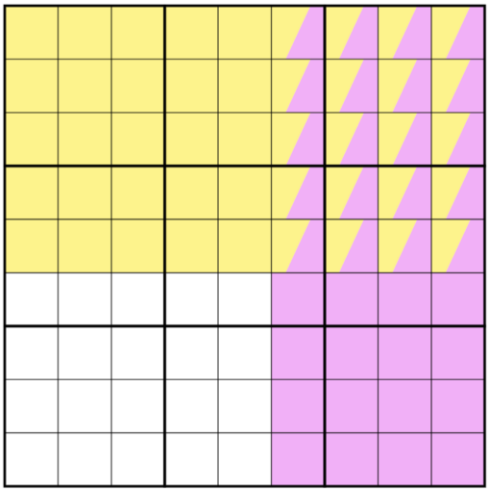
Rules

- Normal sudoku rules apply: Place the digits 1 to 9 once each in every row, column, and 3x3 box
- **Renban**: Digits along a magenta renban line must form a non-repeating consecutive sequence in any order
- **Thermometer**: Digits along a grey thermometer must strictly increase from the bulb end
- **German Whisper**: Adjacent digits along a green line must differ by at least 5
- **Region Sum Line**: Box borders divide a blue line into segments with the same sum

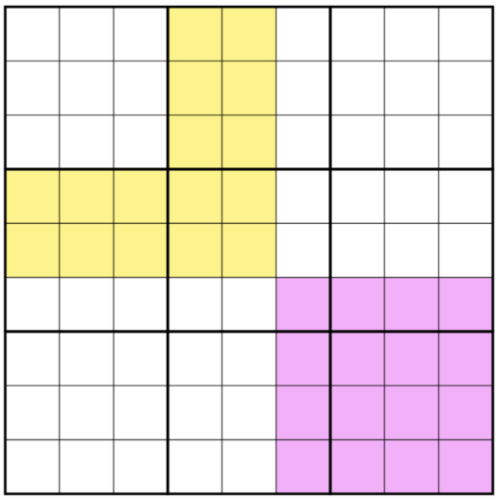
Pre-Requisites

The opening to this puzzle uses Aad's Set Theorem. (Please jump ahead if you are familiar with this)

- By adding and removing sets of digits that we know are a full set of the digits 1 to 9 we can establish relationships between cells in the grid
- Aad's set theorem, which works on any regular 9x9 sudoku, involves comparing rows 1 to 5, against columns 6 to 9. Colour rows 1 to 5 in yellow, colour columns 6 to 9 in pink

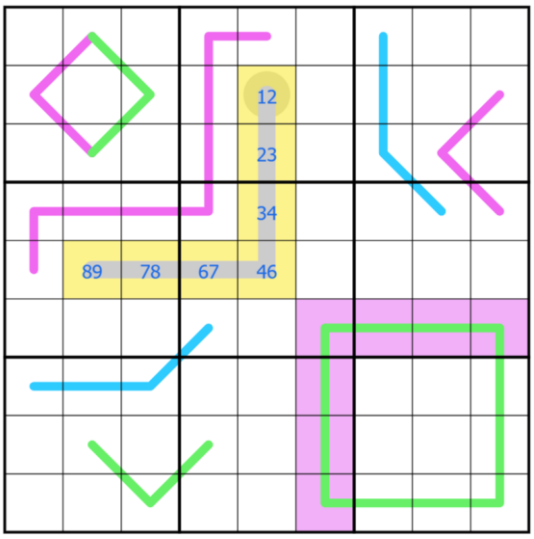


- Yellow is 5 sets of the digits 1 to 9, pink is 4 sets of the digits 1 to 9. We can remove box 1 from yellow and now both yellow and pink are 4 sets of the digits 1 to 9
- We can remove any cells that appear in yellow and pink, and we still know that yellow and pink contain the same digits (although we don't know the exact digits in these sets)

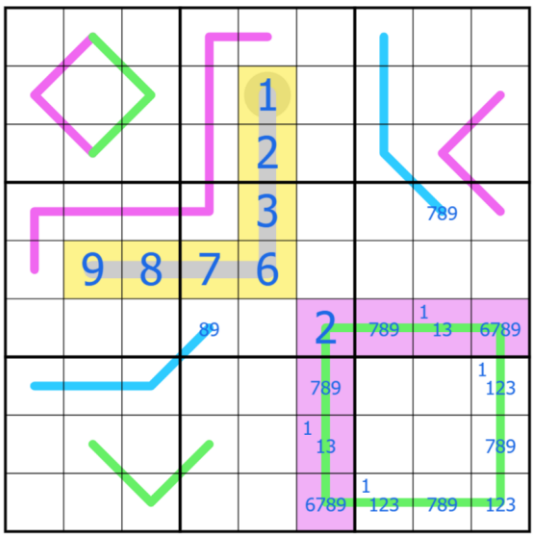


Solve Path

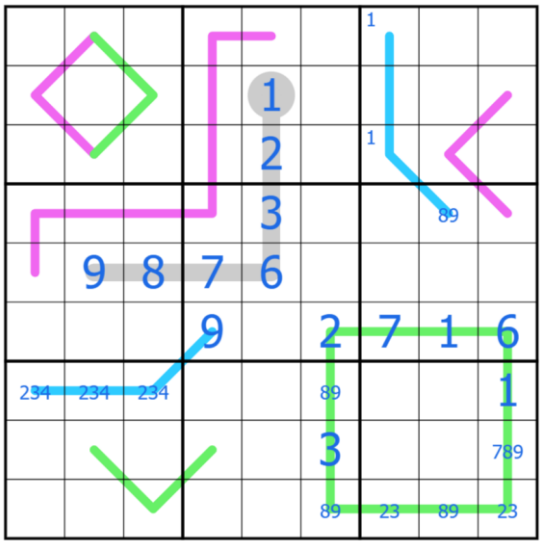
- Use Aad's Set Theorem (above), and also remove from yellow the 9 cell renban and from pink box 9. Yellow and pink now contain 7 cells and these must be the same digits
- Corrolary: The thermometer does not contain a 5
- Corrolary: The German whisper consists of 7 different digits, and it contains at least one or 4 or 6



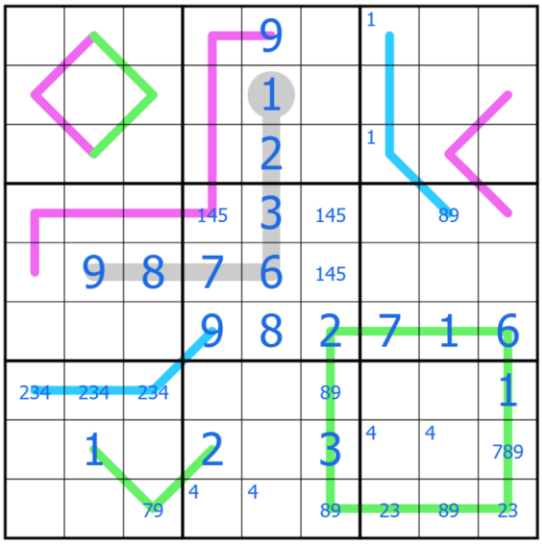
- Where does the 46 go? It can't go in R6C6 because its neighbours must be different by our set, so it must be placed in either R6C9 or R9C6, with two 1s or two 9s as neighbours
- The associated 1 or 9 is pushed on to the region sum line in row 7 or column 7, and 9 can't be placed on a 3 cell region sum line summing to at most 9, so the digit on the thermometer and the whisper is a 6, and we can high/low shade the whisper



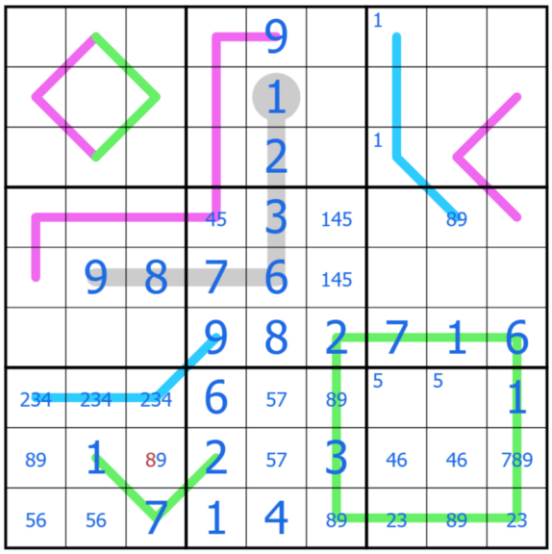
- 3 on a German whisper must be surrounded by 8 and 9, and 3 in R6C8 would break R6C4, so this must be 1 which resolves the 6s on the whisper
- (Alternatively, one of the region sum lines must be 234, so any cells which see both R6C4 and R4C8 can't be 9)



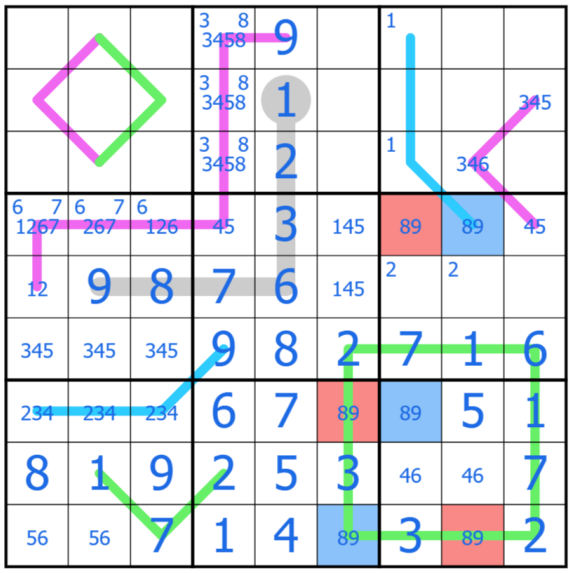
- 9 is placed on the 9 cell renban
- 8 is placed in row 6
- 2 is placed in row 8, which resolves the 3 cell whisper



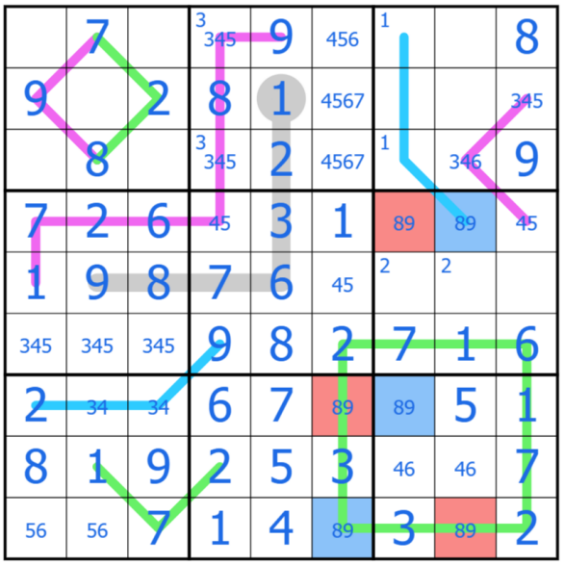
- 4 is placed in column 5
- 89 pair in row 9 does damage



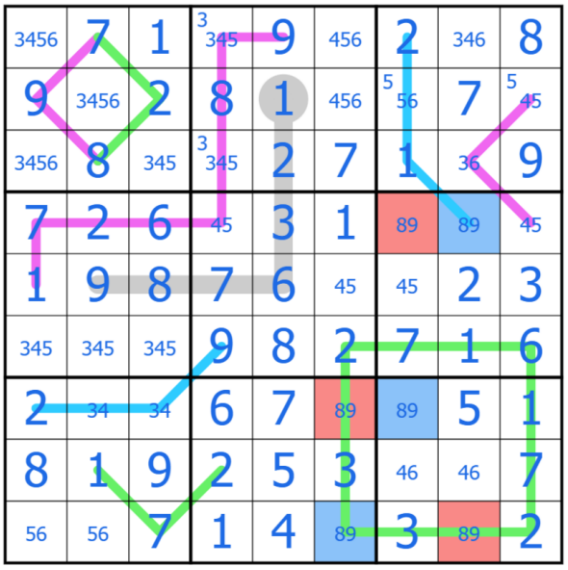
- More digits can be placed in row 789
- Colour the 89 pairs, where is red 89 in column 7? It can't go on the region sum line so is in R4C7
- Makes R4C9 from 45



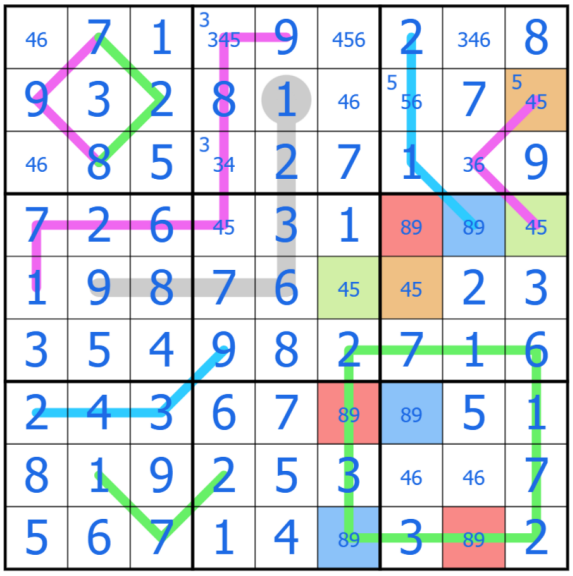
- 45 pair in row 4 makes R4C6 a 1
- 8 and 9 are placed in column 9
- 9 is placed on the renban in box 1, which also fixes the whisper in box 1



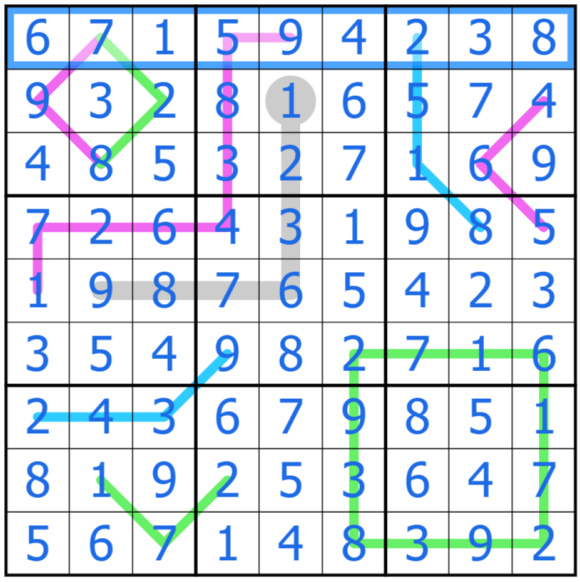
- Region sum line in column 7 can't contain a 3, so it must be 125 or 126
- Places 2 in box 6, along with 3, and constrains the renban in column 9



- 456 triple in row 2 does damage, along with pointing 5s
- Colouring 45 pairs we can show that R2C6 is a 6



- And sudoku to finish


6 7 1 5 9 4 2 3 8
9 3 2 8 1 6 5 7 4
4 8 5 3 2 7 1 6 9
7 2 6 4 3 1 9 8 5
1 9 8 7 6 5 4 2 3
3 5 4 9 8 2 7 1 6
2 4 3 6 7 9 8 5 1
8 1 9 2 5 3 6 4 7
5 6 7 1 4 8 3 9 2