Rainbow Palindromes– Full Solution Guide

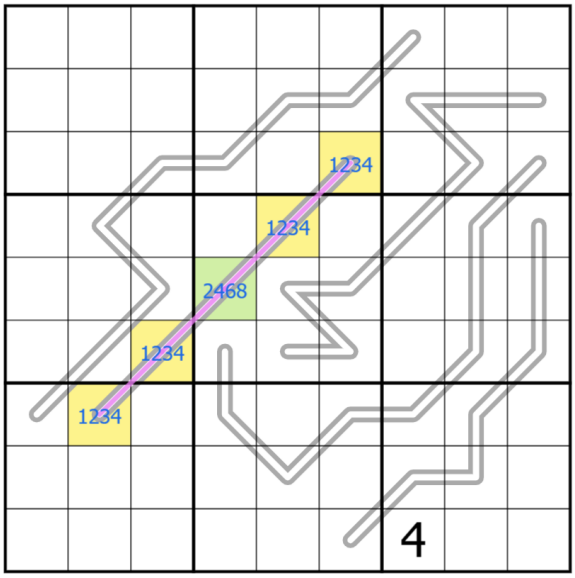
This is a full solution guide to my puzzle *Rainbow Palindromes*, and so spoilers are ahead.

Rules

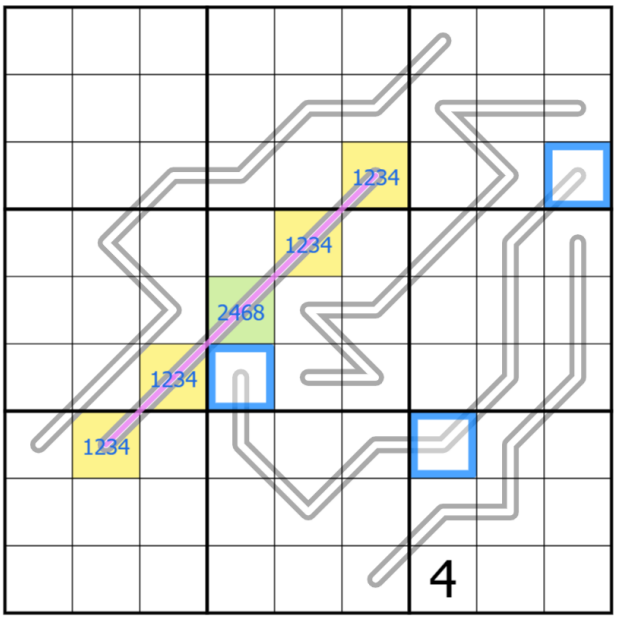
- Normal sudoku rules apply: Place the digits 1 to 9 once each in every row, column, and 3x3 box
- All lines in the grid are palindromes, which must read the same forward as backwards
- In addition, each of the following constraints applies to exactly one line in the grid:
    - **German Whispers**: Adjacent digits must differ by at least 5
    - **Parity**: Adjacent cells must contain one odd and one even digit
    - **Region Sum Line**: Box borders divide the line into segments with the same sum
    - **Ten Line**: Must contain one or more non-overlapping contiguous groups of cells that sum to 10
    - **Zipper**: Digits an equal distance from the center of a zipper line must sum to the digit in the center

Solve Path

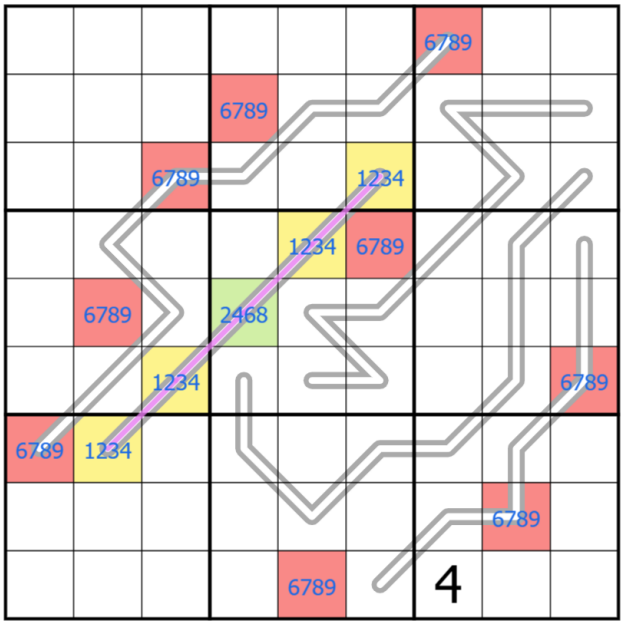
- (NB: The lines have been left hollow to allow these to be filled using the pen tool when they have been established. I will use the standard colours for these, Green = German Whispers; Red = Parity; Blue = Region Sum; Grey = Ten Line; Pink = Zipper)
- There is only one line in the grid capable of being a zipper, which is centered on R5C4, and the center digit is even (green) and the wings contain a digit half the center (yellow)



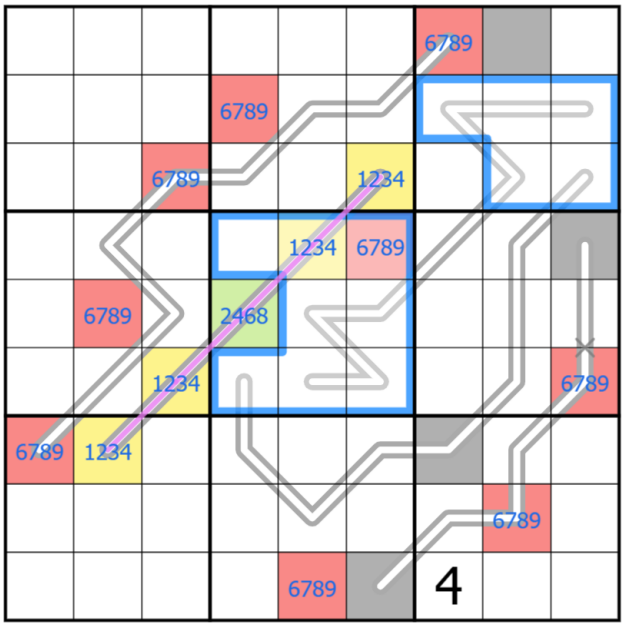
- We need a region sum line. The line centered at R7C8 is invalid for fairly obvious reasons. The line centered at R4C7 is invalid because a 1 cell segment can't join to a 4 cell segment. The line centered at R7C7 is invalid for a slightly more complicated reason. R6C4, R7C7 and R3C9 would all be the same digit by region sum rules, but this digit would have nowhere to go in box 6



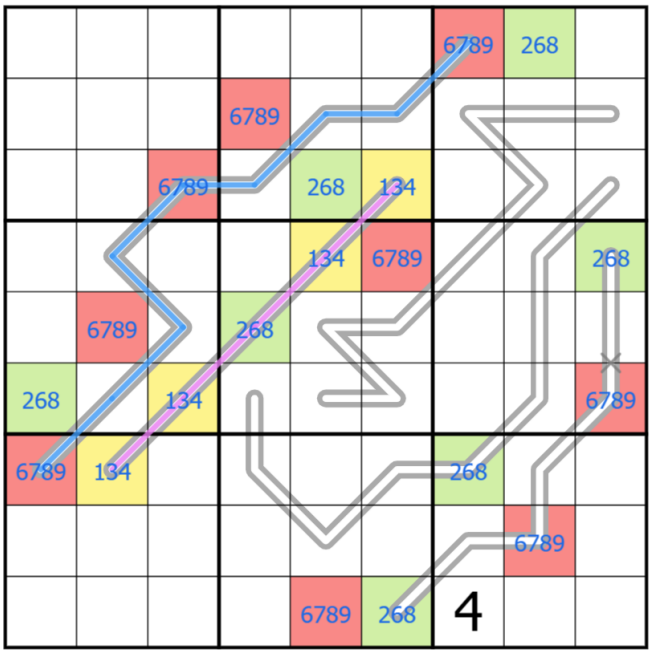
- Hence the line centered at R3C3 is the region sum, this digit (red) is from 6789 and we can place this by sudoku in box 2 and 4
- By palindrome logic we can also place red in the other boxes including in two places on the line centered at R7C8
- Meta: The line centered at R7C8 is not the ten line. With two high digits next to the central digit, there is no way to make valid sums to 10



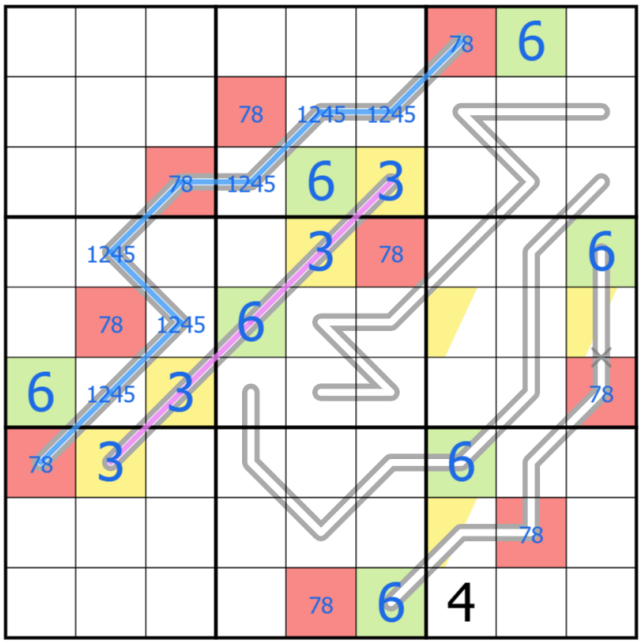
- Meta: The line centered on R7C8 is either German Whisper or Parity. Regardless, this line will oscillate between two families of values
- Consider R9C6 in box 9. By sudoku and palindrome it is limited to R7C7 or R7C8. However, R7C8 is three steps away from itself on the line, and this is invalid, so this is placed in R7C7
- By sudoku and palindrome this can be placed in column 8



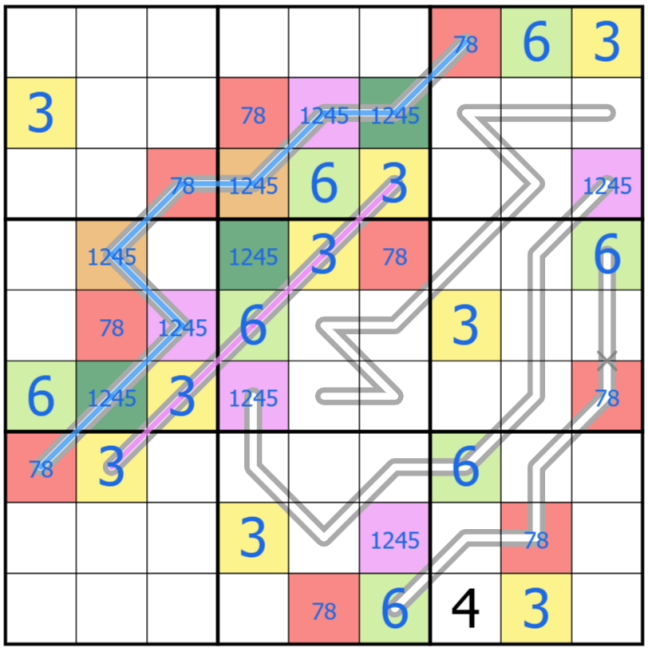
- In box 5 this digit eliminated from all cells by sudoku or palindrome other than R5C4, so we learn that this digit is green and we can place more greens
- And hence green is not 4 so yellow is not 2



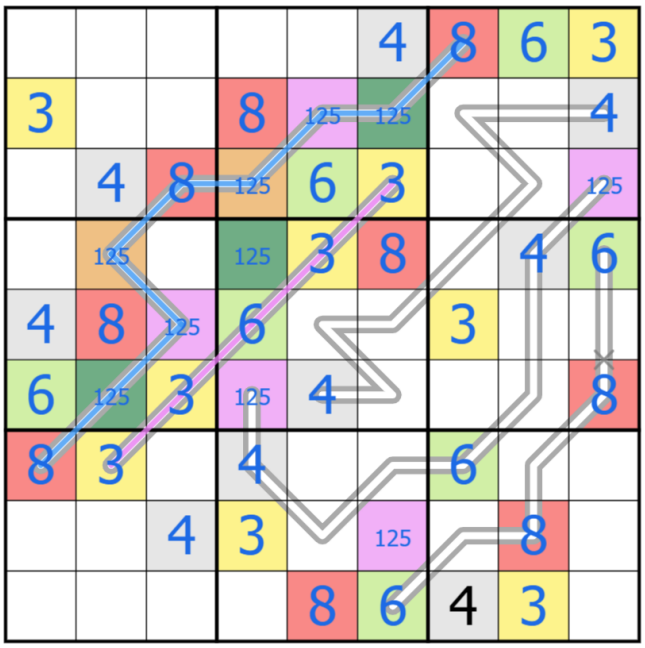
- Yellow in row 5 is either R5C7 or R5C9 which then palindromes to R8C7
- Either way yellow sees the given 4, so this is not 4, and green is not 8
- In addition, a 3 cell region sum line will use 1 or 2 (or both) so by box 4 green can't be 2 with yellow as 1, so we learn that green is 6 and yellow is 3
- A 3 cell sum to 9 uses either 3 or 6, so red is from 78 and its digits are 124 or 125



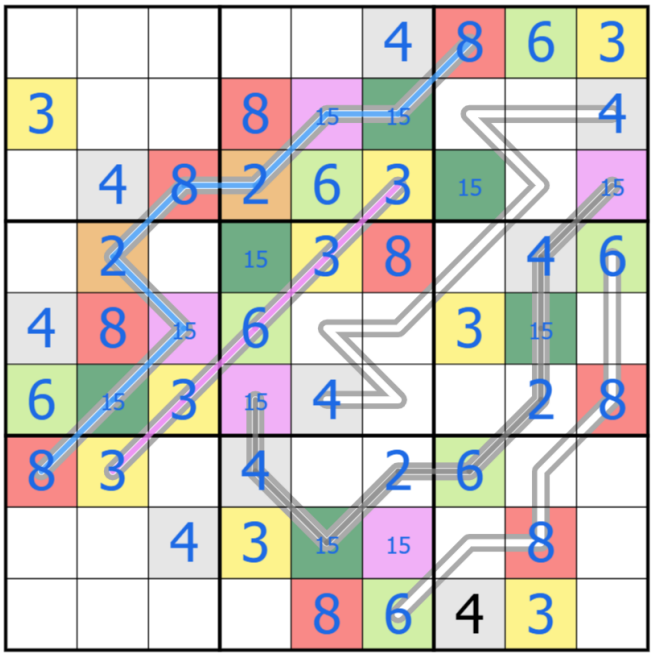
- We can colour the 1245. R6C2 (dark green) can be placed in box 5. R5C3 (purple) can be placed in box 5, 8 and 3
- We can also map some 3s around the grid



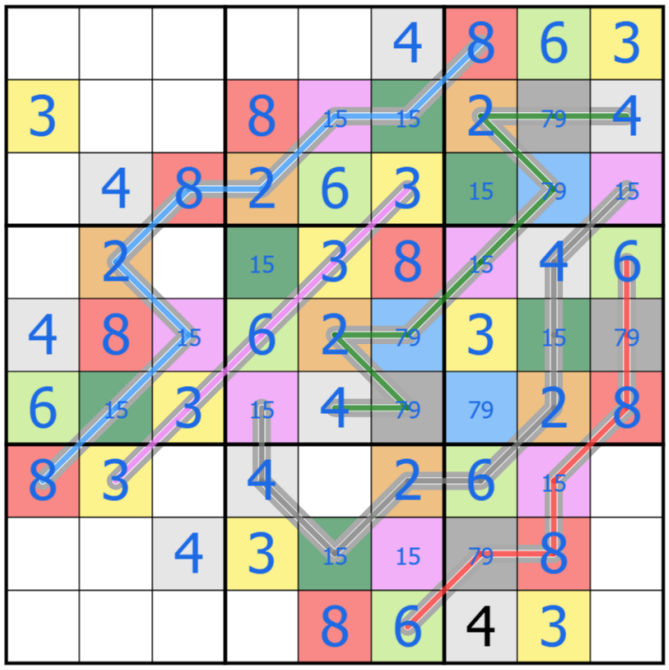
- Consider R7C4. By sudoku and palindrome we can map this into every box, and learn that this is R9C7, and is thus a 4
- This leaves only 125 for the region sum line making red 8



- We now have 4 and 6 three steps apart on the line centered at R7C7. This can't be German Whisper or Parity, so must be Ten Line
- The only way to make the 6 work on a Ten Line is with a pair of 2s next to it
- The other two segments are 145



- The line centered at R7C8 must be Parity, as this has a 6 next a cell which can't be 1 or 5 by the 15 pair
- Hence the line centered at R4C7 must be German Whisper
- We can map more cells around the grid and complete the lines with candidates



- By whisper we can resolve the 15 and 79 pairs
- And then sudoku to finish