

Hilbert Hotel– Full Solution Guide

This is a full solution guide to my puzzle *Hilbert Hotel*, and so spoilers are ahead.

Rules

- Normal sudoku rules apply: Place the digits 1 to 9 once each in every row, column, and 3x3 box
- **Cipher:** The letters A to I represent distinct digits from 1 to 9. Which letter corresponds to which digit is to be determined by the solver
- **Numbered Rooms:** Clues outside the grid are numbered rooms, and give the digit to be placed in the Nth position in that row or column, where N is the digit in the first position
- **Clones:** Identical grey shapes are clones, where the same digits appear in the same position. (Clones have been marked with cages for ease of visibility, but digits may repeat in a cage)

Solve Path

- A couple of secrets of numbered rooms
 - The first digit next to a numbered room clue can't be 1 or the clued digit, unless the clue is itself a 1
 - Two identical clues at either end of a row or column will have first digits that sum to 10. For example $R1C1 + R1C9 = 10$
- Can R1C1 be 1, and A is a 1? No, because this puts 9 in R1C9, 1 in R9C9, but I can't be 1 breaking the numbered room. Symmetrically in R1C9
- R1C1 can't be 2 or 3 as this would put A twice in the same box, and can't be 5 as R1C9 would also be a 5
- This applies symmetrically to R1C9 so these two are a 46 pair
- This places A in front of one of the C clues in R1, and one each of the E clues in C1 and C9
- A in R5 is in box 5

	A	B		C		C		C		A	
A	46			A		A				46	A
D	A									A	D
D	A									A	D
G											B
C											H
	C		G	I	A	H		I			I

A	
B	
C	
D	
E	
F	
G	
H	
I	

- Can A be 1? No, as this would break the E and C clues
- Can A be 9? No, as this would place C into row 9 of box 8, but this would make R9C1 a 46 breaking the C1 clues looking at each other
- Can A be 3 or 7? No, as by clone logic this would place the same digit into column 3 or 7, like so

A	B		C		C		C	A	
A	46		A		A			46	A
D	A							A	D
D	A							A	D
G									
C									
	C		G	I	A		H	I	

A	
B	
C	
D	
E	
F	
G	
H	
I	

- Hence A is 2 or 8, putting 28 pairs in box 4 and box 6. I have arbitrarily assigned A as purple, the other digit from 28 as grey
- This means that D is in R4C2, R6C8, or R4C8, R6C2

	A	B		C	C	C	A		
A	46			A		A		46	A
D	A ₂₈	D					D	A ₂₈	D
		E	A	A	A	F			
D	A ₂₈	D					D	A ₂₈	D
									B
G									H
C									I
	C	G	I	A	H	I			

A	28
B	
C	
D	
E	
F	
G	
H	
I	

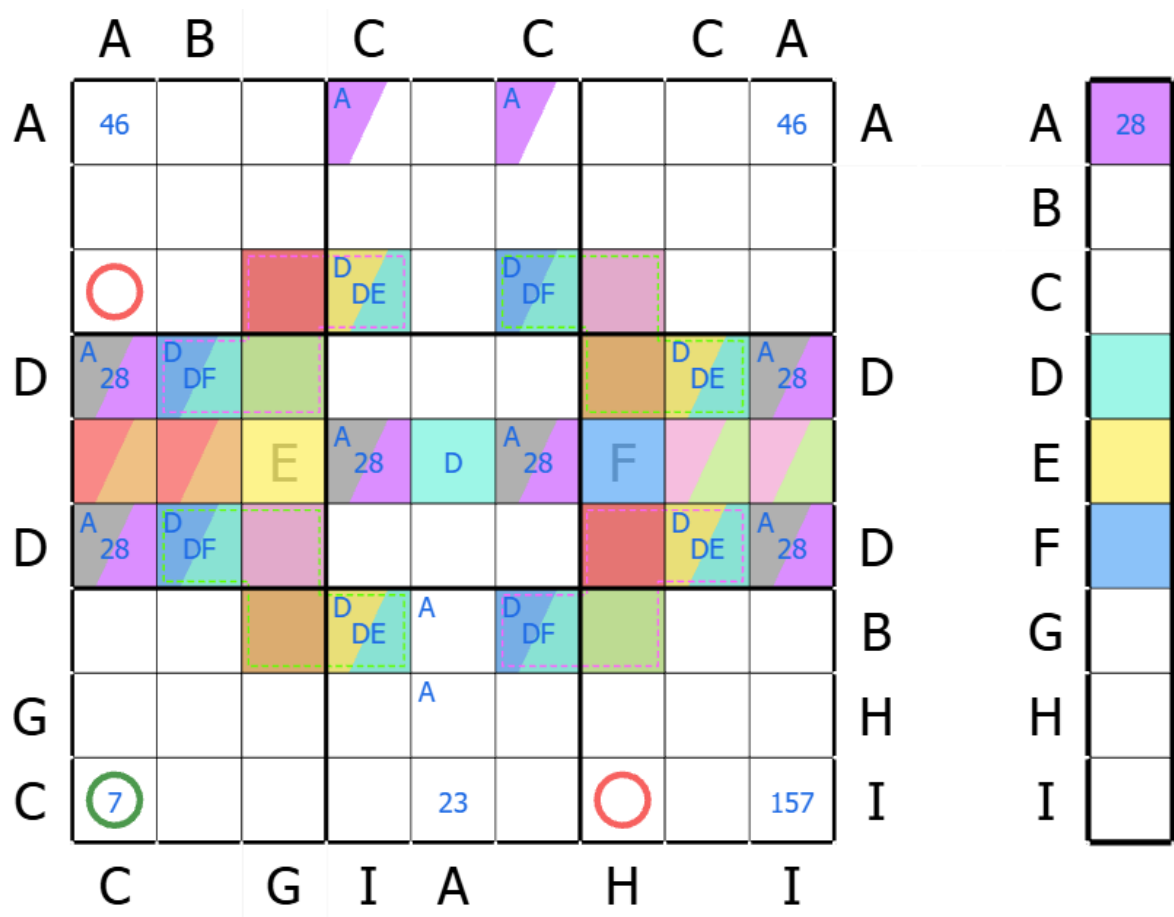
- Regardless of which way around the D goes in the clones, R5C5 is always D. I have arbitrarily assigned this the colour cyan
- Places a 28 pair in R5 in box 5
- Colour R4C3 green. By clone it goes in R7C7, then in box 6 goes in R5C89
- Similarly, R6C3 (pink) = R5C89
- R4C7 (orange) and R7C7 (red) make orange/red pair in R5C12
- Arbitrarily assign E and F as yellow and blue

	A	B		C	C	C	A		
A	46			A	A			46	A
				D	D				
D	A 28	D					D	A 28	D
D	A 28	D					D	A 28	D
G									B
C									H
	C		G	I	A	H	I		I

- Box 6 and by clone column 4 has a DE pair
- Similarly box 4 and column 6 has a DF pair
- R9C1 is limited. 28 ruled out by sudoku, 46 ruled out as it would break the C1 clues. 3 ruled out as it would place 2 Cs in the same box. So limited to 1579, and R9C9 similarly
- A in column 5 is in box 8, and can't go in R9C5
- Makes R9C5 from 23

	A	B		C	C		C	A		
A	46			A		A			46	A
				D		D				
D	A	D					D	A		D
D	A	D					D	A		D
G										B
C	1579				23				1579	H
	C		G	I	A		H	I		I

- Can the C in box 5 be indexed by the C4/6 clues? No, as R1 has a 46 pair, and C is not from 28 by C1, so the C clues in C4/6 are indexing cells in box 2 and box 8
- R9C1 can't be 5 as the C in column 5 is in box 5
- R9C1 and R9C9 also can't be 9 as this would make the digits in the opposite corners the same but R1C1 and R9C9 are definitely different, and the other diagonal similarly
- So R9C1 down to 17, and R9C9 down to 157
- What happens if R9C1 is 7?...



- C would be placed in the red circles. However, these see seven different colours, only excluding cyan and yellow. As these are D and E, this would leave no options available for the digit C
- Makes R9C1 a 1, and C is 1
- R9C9 similarly can't be 7, so this is 5
- Makes R9C5 and R5C9 the same digit, and by available options this can only make I a 3

	A	B		C	C	C	A		
A	46			⁸ ₁₂₈ A	⁸ ₁₂₈ A			46	A
D	^A ₂₈	^D _{DF}			1		^D _{DE}	^A ₂₈	D
D	^A ₂₈	^D _{DF}			1		^D _{DE}	^A ₂₈	D
G				1		1			B
C	1				3			5	H
	C		G	I	A	H	I		I

- 1 in R8 is limited, and 1 in R7 is limited
- R7C9 can't be 1 as B is not 1. Hence 1 in column 9 is in box 3
- R1C8 is limited to 57 by sudoku and available spots for 1 in the column
- Leaves only one place for 3 in row 1
- Places B in R3C2

	A	B		C		C		C		A	
A	46	3		8 ^A ₁₂₈		8 ^A ₁₂₈		57	46	A	
									1		
		B		D _{DE}		D _{DF}			1		
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D	
			E	A ₂₈	D	A ₂₈	F		3		
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D	
	B		B	D _{DE}	A	D _{DF}	1	1	79	B	
G				1		1				H	
C	1				3				5	I	
	C		G	I	A		H		I		

A	28
B	
C	1
D	
E	
F	
G	
H	
I	3

A	28
B	
C	1
D	
E	
F	
G	
H	
I	3

- By sudoku and indexing R7C9 is from 79
- If R7C1 were B, then by sudoku B would be either green or pink in box 4, but B sees these in rows 3 and 7 already
- Makes R7C3 B, and then R5C1
- Hence R7C9 is 7

	A	B		C		C		C		A	
A	46	3		8 128 A		8 128 A		57	46		A
									1		
		B		D DE		D DF			1		
D	A 28	D DF	3		1			D DE	A 28		D
	B		E	A 28	D	A 28	F		3		
D	A 28	D DF	3		1			D DE	A 28		D
			B	D DE	A	D DF	1	1	7		B
G				1		1					H
C	1				3				5		I
	C		G	I	A		H		I		

A	28
B	
C	1
D	
E	
F	
G	
H	
I	3

A	28
B	
C	1
D	
E	
F	
G	
H	
I	3

- A is placed in row 3
- Places 2 in row 3 in box 2
- Makes a 18 pair in box 2
- Makes A an 8

	A	B		C		C		C	A	
A	46	3		18		18		57	46	A
		8	8						1	
		B		D DE	2	D DF		8	1	
D	A 28	D DF	3		1			D DE	A 28	D
	B		E	A 28	D	A 28	F		3	
D	A 28	D DF	3		1			D DE	A 28	D
			B	D DE	8	D DF	1	1	7	B
G				1		1	8			H
C	1				3		8		5	I
	C		G	I	A	H	I			

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

- 3 in row 3 is placed in R3C7, and is pink
- Allows placing more 3s in pink or by sudoku

	A	B		C		C		C	A	
A	46	3		18		18		57	46	A
		8	8	3		3			1	
		B		D DE	2	D DF	3	8	1	
D	A 28	D DF		3	1	3		D DE	A 28	D
	B		E	A 28	D	A 28	F		3	
D	A 28	D DF	3		1			D DE	A 28	D
	3		B	D DE	8	D DF	1	1 3	7	B
G	3			1		1	8	3		H
C	1				3		8		5	I
	C		G	I	A		H		I	

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

- R9C4 is limited. It is only 68 as it needs to index 3 in the column
- As we have 8 in the box, R9C4 is only 6, indexing 3 in box 5 and 2

	A	B		C		C		C		A	
A	46	3		18		18		57	46	A	
		8	8			3			1		
		B		D _{DE}	2	D _{DF}	3	8	1		
D	A ₂₈	D _{DF}		3	1			D _{DE}	A ₂₈	D	
	B		E	A ₂₈	D	A ₂₈	F		3		
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D	
	3		B	D _{DE}	8	D _{DF}	1	1	3	7	
G	3			1		1	8	3			
C	1			6	3		8		5		
	C		G	I	A		H		I		

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

- C, G and H are limited to the colours red, green and grey
- We can mark the options for R9C3 and R9C7 with this in mind

	A	B		C		C		C	A	
A	46	3		18		18		57	46	A
		8	8			3			1	
		B		D DE	2	D DF	3	8	1	
D	A 28	D DF		3	1			D DE	A 28	D
	B		E	A 28	D	A 28	F		3	
D	A 28	D DF	3		1			D DE	A 28	D
	3		B	D DE	8	D DF	1	1 3	7	B
G	3			1		1	8	3		H
C	1		2789	6	3		8 489		5	I
	C		G	I	A		H		I	

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

- By indexing options, neither G nor H can be green
- So 1 is green, allowing more 1s to be placed including in box 1
- Places 8 and 2 in box 1
- Gives R1C8 by indexing
- Reduces R9C3 to 79

	A	B		C		C		C	A	
A	46	3	2	18		18		5	46	A
		1	8			3				
		B		D _{DE}	2	D _{DF}	3	8	1	
D	A ₂₈	D _{DF}	1	3	1			D _{DE}	A ₂₈	D
	B		E	A ₂₈	D	A ₂₈	F	1	3	
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D
	3		B	D _{DE}	8	D _{DF}	1	3	7	B
G	3			1		1	8	3		H
C	1		79	6	3		8 ₄₈₉		5	I
	C		G	I	A		H		I	

A	8
B	
C	1
D	
E	
F	
G	
H	
I	3

- By indexing R8C1 isn't 3, placing the last 3s
- Red in box 7 is in R8C1
- This removes the option for D to be red, so D is grey (2) and H is red
- Completes box 9 by indexing and sudoku

	A	B		C		C		C	A	
A	46	3	2	18		18		5	46	A
		1	8			3				
		B		D _{DE}	2	D _{DF}	3	8	1	
D	A ₂₈	D _{DF}	1	3	1			D _{DE}	A ₂₈	D
	B		E	A ₂₈	D	A ₂₈	F	1	3	
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D
	3		B	D _{DE}	8	D _{DF}	1	6	7	B
G				1		1	8	3	9	H
C	1		9	6	3		4	2	5	I
	C		G	I	A		H		I	

A	8
B	
C	1
D	
E	
F	
G	2
H	
I	3

- 2 in row 8 is limited to two spots, making a 12 pair
- Makes R8C1 a 46 by indexing
- Gives a 46 pair in C1

	A	B		C		C		C	A		
A	46	3	2	18		18		5	46	A	A
		1	8			3					B
		B		D _{DE}	2	D _{DF}	3	8	1		C
D	A ₂₈	D _{DF}	1	3				D _{DE}	A ₂₈	D	D
	B		E	A ₂₈	D	A ₂₈	F	1	3		E
D	A ₂₈	D _{DF}	3		1			D _{DE}	A ₂₈	D	F
	3	2	B	D _{DE}	8	D _{DF}	1	6	7	B	G
G	46			12		12	8	3	9	H	H
C	1	8	9	6	3		4	2	5	I	I
	C		G	I	A		H		I		

- Red sees a 4 in C7, so red (H) is 6
- Makes R1C1 a 4
- Resolves the 18 pair in box 2, the 28 pairs in boxes 4, 5 and 6, and the 12 pair in box 8 by indexing

	A	B		C		C		C	A	
A	4	3	2	8	79	1	79	5	6	A
	579	1	8		6	3				
	579	B	6	^D DE	2	^D DF	3	8	1	
D	^A ₂₈	^D DF	1	3		6		^D DE	^A ₂₈	D
	B	6	E	2	D	8	F	1	3	
D	^A ₂₈	^D DF	3		1		6	^D DE	^A ₂₈	D
	3	2	B	^D DE	8	^D DF	1	6	7	B
G	6	457	457	1	45	2	8	3	9	H
C	1	8	9	6	3	7	4	2	5	I
	C		G	I	A		H		I	

A	8
B	
C	1
D	
E	
F	
G	2
H	6
I	3

- B sees 4, 7 and 9 in C1, C3, R7, so this is 5

	A	B		C		C		C	A	
A	4	3	2	8	79	1	79	5	6	A
	79	1	8	5	6	3				
	79	5	6	^D _{DE} 2	^D _{DF}	3	8	1		
D	^A ₂₈	^D _{DF}	1	3		6	5	^D _{DE}	^A ₂₈	D
	5	6	E	2	^D	8	F	1	3	
D	^A ₂₈	^D _{DF}	3		1	5	6	^D _{DE}	^A ₂₈	D
	3	2	5	^D _{DE}	8	^D _{DF}	1	6	7	B
G	6	47	47	1	5	2	8	3	9	H
C	1	8	9	6	3	7	4	2	5	I
	C		G	I	A		H		I	

A	8
B	5
C	1
D	479
E	479
F	479
G	2
H	6
I	3

- Lots of sudoku follows
- DEF are down to a 479 triple
- F not 4 by column 7, E not 9 by column 3

	A	B		C		C		C	A		
A	4	3	2	8	79	1	79	5	6	A	A
	79	1	8	5	6	3	2	79	4		B
	79	5	6	479	2	49	3	8	1		C
D	A ₂₈	479	1	3	479	6	5	479	A ₂₈	D	D
	5	6	47	2	D ₄₇₉	8	79	1	3		E
D	A ₂₈	479	3	479	1	5	6	479	A ₂₈	D	F
	3	2	5	479	8	49	1	6	7	B	G
G	6	47	47	1	5	2	8	3	9	H	H
C	1	8	9	6	3	7	4	2	5	I	I
	C		G	I	A		H		I		

- By column 6, yellow (E) is 7
- Makes blue 9
- Makes cyan 4
- Remaining 479s can be resolved. Don't forget about the clones as they are probably completely obscured with colours now!

	A	B		C		C		C	A		
A	4	3	2	8	9	1	7	5	6	A	A
	7	1	8	5	6	3	2	9	4		
	9	5	6	7	2	4	3	8	1		
D	^A ₂₈	9	1	3	7	6	5	4	^A ₂₈	D	D
	5	6	7	2	4	8	9	1	3		
D	^A ₂₈	4	3	9	1	5	6	7	^A ₂₈	D	D
	3	2	5	4	8	9	1	6	7		B
G	6	7	4	1	5	2	8	3	9		H
C	1	8	9	6	3	7	4	2	5		I
	C		G	I	A		H		I		

A	8
B	5
C	1
D	4
E	7
F	9
G	2
H	6
I	3

- Remaining 28s resolved by the A clues in C1, C9, or the D clues in R4, R6

	A	B		C		C		C	A			
A	4	3	2	8	9	1	7	5	6	A	A	8
	7	1	8	5	6	3	2	9	4		B	5
	9	5	6	7	2	4	3	8	1		C	1
D	8	9	1	3	7	6	5	4	2	D	D	4
	5	6	7	2	4	8	9	1	3		E	7
D	2	4	3	9	1	5	6	7	8	D	F	9
	3	2	5	4	8	9	1	6	7	B	G	2
G	6	7	4	1	5	2	8	3	9	H	H	6
C	1	8	9	6	3	7	4	2	5	I	I	3
	C		G	I	A		H		I			