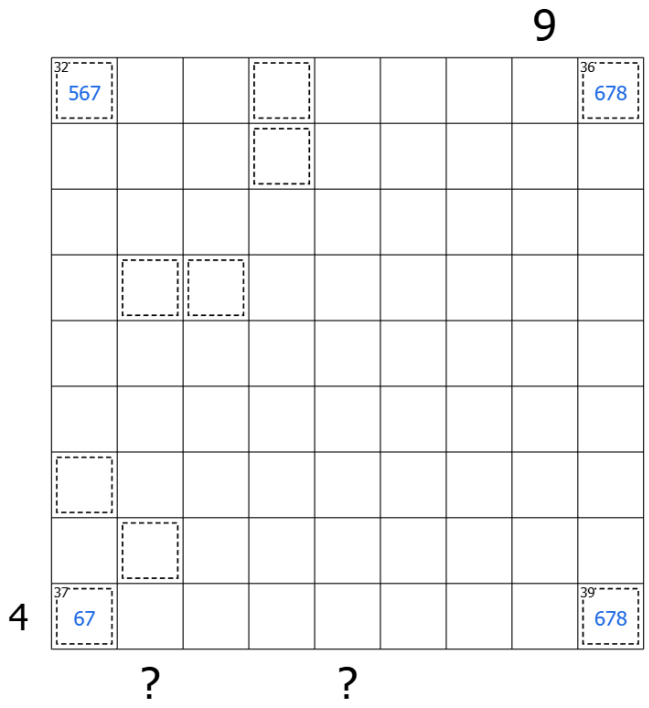
One Of Two – Solve Path

By MantaRay

1. To begin the puzzle, pencil-mark the 4 corner cages with their possible values. The main thing of note here is that all 4 corners must be part of a different cage – and each of these regions must have at least 1 cell that is not on the grid edge (as none of the corners can be a 9).



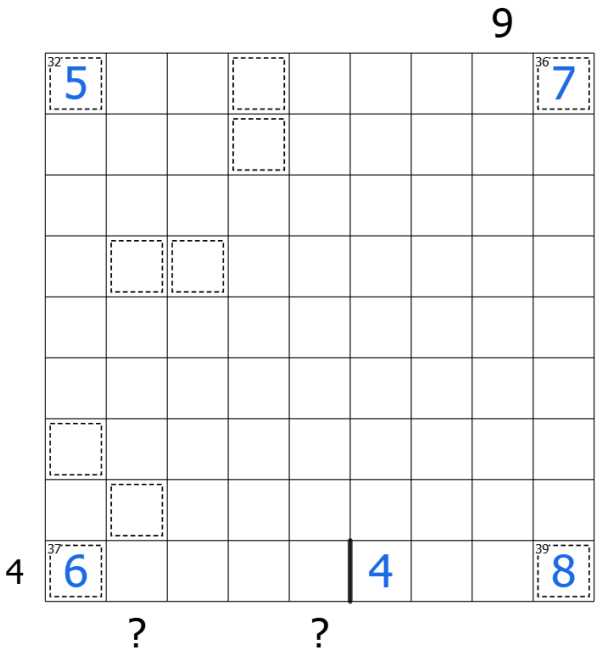
2. The next point to note is that the digits in the 4 corners only appear (up to) 3 times around the grid border. However, there are 4 regions that cover the grid border, and meaning that the corner digits must all exist in one of the 'unseen' cells of the other 3 corner regions (i.e. the cells not on the grid edge).

3. The first question to be asked is "can the corner cells contain any repeated digits?". Let's first analyse R1C9 and R9C1, which could be double 6 or double 7. Double 7 is impossible, as the region containing R9C9 must 'see' cells that sum to 39 but cannot 'see' a 7 – meaning the region would contain digits that sum to at least 46.

4. If R1C9 and R9C1 both contained a 6, then R9C9 would need to be an 8. However, given that the region containing R1C1 would be 'missing' a 6 (taking its running total to 38 – the 'seen' cells summing to 32 and the extra 'unseen' 6), which of the corner regions would contain the 'unseen' 8 that has been placed in R9C9? R1C9 contains a 6 (meaning it has 3 'unseen' cells that sum to 8), R9C1 also contains a 6 (meaning it has 3 'unseen' cells that sum to 9) and the region containing R1C1 has already been allocated cells that sum to 38 and does not include an 'unseen' 8 – therefore, no corner region can house another 8.

5. It has now been shown that R1C9 and R9C1 cannot contain the same digit. Let's now perform the same procedure for R1C1 and R9C9. Again, these could either be double 6 or double 7. Again, double 7 does not work because R9C1 would be a 6 and its region would have 3 'unseen' cells that sum to 8 but must contain a 7.

6. If R1C1 and R9C9 both contained a 6, then R9C1 must contain a 7 and R1C9 must contain an 8 (as it was previously proven that R1C9 and R9C1 are different). But the region containing R1C9 must now only have one 'unseen' cell that contains a 9, so where does the 6 go in that region?



7. It can now be said that all 4 corner cells must contain a different digit, making R1C1 a 5. This now means that the region containing R1C1 has 4 'unseen' cells that sum to 13 – meaning it cannot house the 8 that must exist in R1/9C9. In fact, the only configuration of digits in the corners that could facilitate the required 8 is if R1C9 was a 7 (which would have 2 'unseen' digits summing to 9, which must be a 1 and an 8). This finishes the corner digits. A 4 can also be placed in R9C6 (along with a region border) thanks to R9C1.

8. The total value of the digits accounted for in the 4 corner regions can now be determined, remembering that the 4 corner digits must themselves appear in the 'unseen' cells of these regions. The total value allocated to the 4 corner regions so far is now:

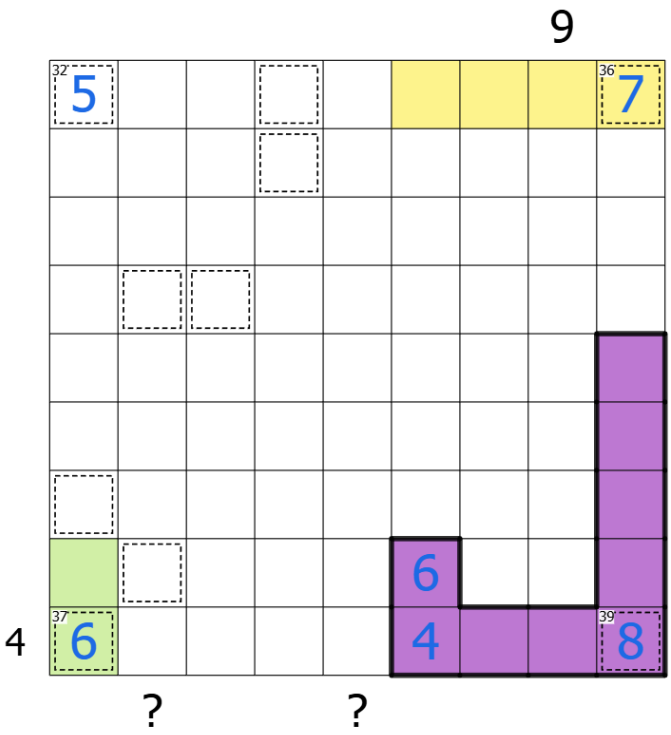
$$32 + 36 + 37 + 39 + (5 + 6 + 7 + 8) = 170$$

9. There are 26 'seen' cells in the 4 corner regions, leaving 10 'unseen'. 4 of these are allocated to the digits in the corner cells, meaning that the 6 remaining 'unseen' cells must sum to 10 (as all 4 corner regions must sum in total to 180). However, note that none of the 4 corner regions can contain more than one of each corner digit. Therefore, 3 of the 'unseen' digits exist within the R1C1 region, 2 are within the R9C1 region and 1 is in the R1C9 region. The only possible way to have 3 different digits add to 2 different digits add to another digit to equal 10 is:

$$(1 + 2 + 3) + (1 + 2) + 1$$

10. Therefore, the R1C1 region has 'unseen' cells with the digits {1, 2, 3, 7}, R1C9 has an 'unseen' {1, 8} pair, R9C1 has an 'unseen' {1, 2, 5} triple and R9C9 has an 'unseen' 6. What's more, the 4 corner cells can only 'see' 26 of the 32 grid border cells. This means that the other 6 grid border cells must be the non-corner digits that are 'unseen' by the corners – three 1s, two 2s and a 3.

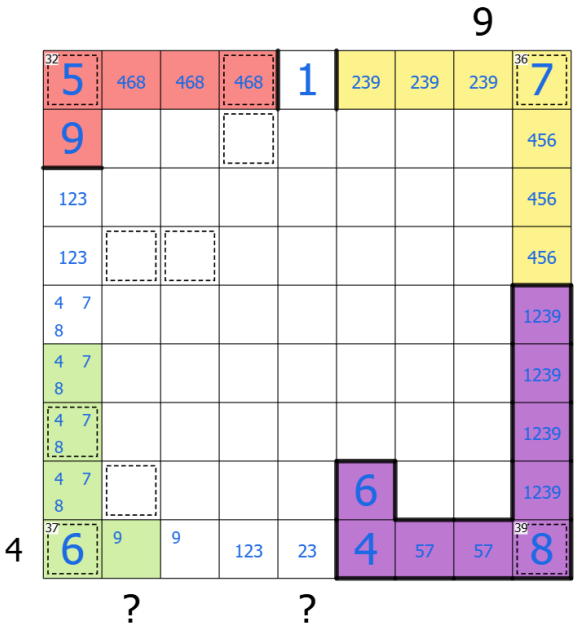
11. The corner regions can now start to be marked in. R9C9 can 'see' a maximum of 4 cells along row 9, so must 'see' up to at least R5C9. This then forces R9C1 up to R1C6. Similarly, R9C1 must also 'see' R8C1.



12. The 4 in R9C6 cannot be 'unseen' by a corner region, so must be part of the R9C9 region. R9C6 does not have a cage, meaning that it cannot 'see' 4 cells – it must therefore also 'see' R8C6 (which must contain the missing 6) and completes the R9C9 region.

13. The 5 in the R1C9 region (which must be 'seen') lives in column 9. Similarly, the 6 in the R1C1 region must live in row 1 – which pushes the 6 in the R1C9 region into column 9 as well. Finally, the 4 in the R9C1 region must live in column 1, meaning that the 4 in the R1C1 region must live in row 1 and the 4 in the R1C9 region must live in column 9. However, this completes the 'seen' section of the R1C9 region! What's more, there are now no 'unseen' cells in column 9 which means that the other 3 borders (i.e. column 1 and rows 1/9) must all have at least 1 'unseen' cell (as there are three 'unseen' 1s around the border).

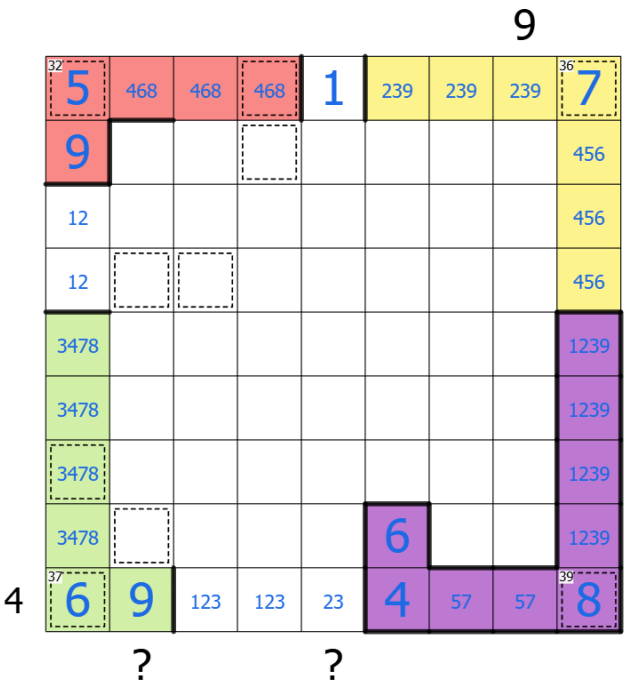
14. Column 9 can now be fully pencil-marked, along with R9C7/8 (which must be a 5-7 pair). The 8 that exists in the R9C1 region must be in column 1, meaning that the 8 that exists in the R1C1 region must be in row 1. However, this means that the R1C1 region now has a 4-6-8 triple in row 1, but must leave at least 1 'unseen' cell – therefore the 'seen' section of the R1C1 region can be marked and R1C5 must be a 1.



15. R1C8 cannot be a 9 (due to the Numbered Rooms clue) and R2C1 must be a 9 (for the Say What You See cage total). As there are at most 3 'unseen' cells in column 1 and row 9, the R9C1 region can be partially coloured in.

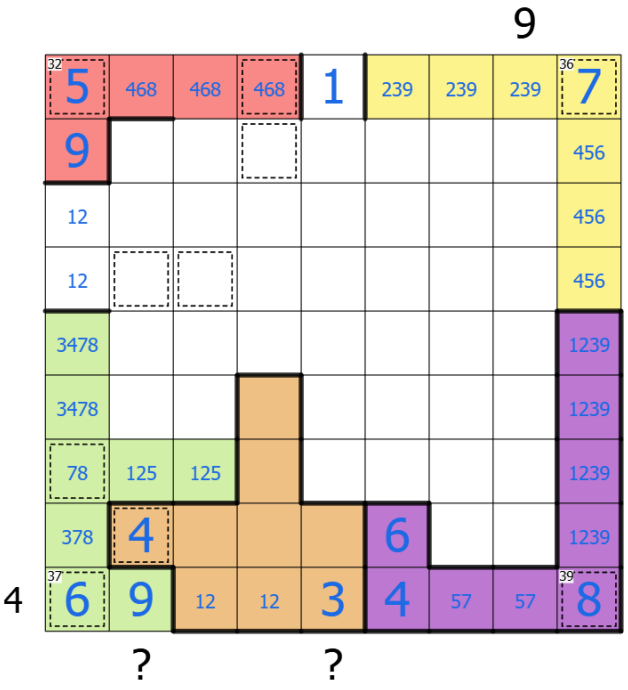
16. If the R9C1 region took R9C3, then there would be 3 'unseen' cells in column 1 (which would need to contain a 1-2-3 triple). This would leave an 'unseen' 1-2 pair for row 9, making R9C5 a 2. But this is broken – R8C5 must be in a different region to R9C5 to satisfy the Numbered Rooms constraint, but then R9C5 would only 'see' two cells (itself and R9C4) which would require a cage in the cell.

17. R9C1s 'seen' cells can now be coloured, and R9C2 must be a 9 (which also creates a border between R1/2C2). The grid border can now be fully pencil-marked.



18. R7C1 must be a 7 or 8, meaning at least 2 more cells must be 'seen'. This then means that R8C2 cannot be part of the green region, as it would be the 9th cell but only 'sees' 4 cells. It must therefore be part of a new region, and contain a 2, 3 or 4.

19. If R8C2 was a 2 then it could only 'see' two cells and must drop into R9C3, picking up all three uncoloured digits – one of which is another 2. R8C2 is therefore a 3 or 4.



20. R8C2 now must 'see' at least 3 cells horizontally, so how could R9C5 be part of a different region to R8C2? As a Numbered Room clue it cannot escape up column 5 without being blocked in by a new region – it must therefore belong to the same region as R8C2 (along with R9C3/4), making R8C2 a 4 and R9C5 a 3. R9C5 being a 3 forces its region to escape up column 4, completing its region and making R7C1 a 7.

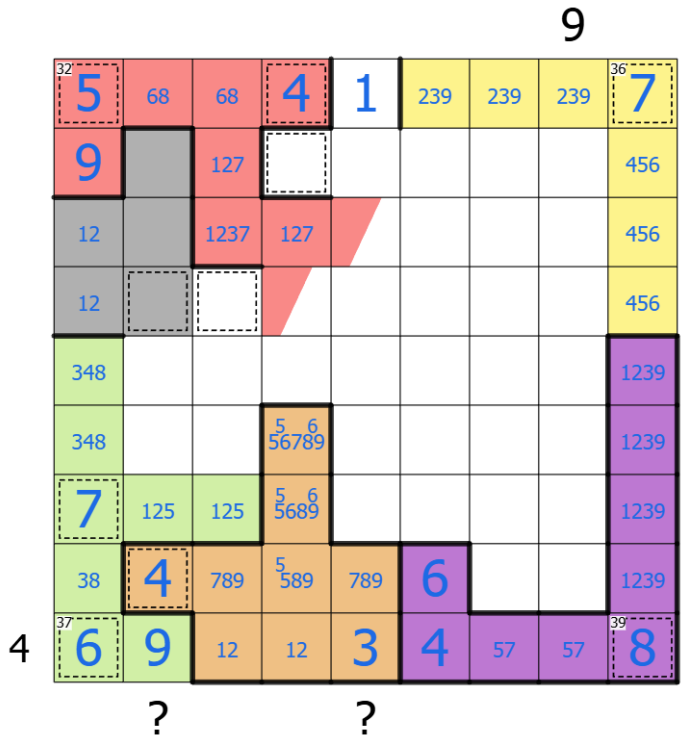
21. The orange region can now be fully pencil-marked (noting the 6 in R8C6 and the lack of cages in the remaining orange cells). R1C4 can no longer be a 6.

22. If R1C4 was an 8, then it would close off the empty region on in columns 1/2/3 – but there are an incorrect number of cells left to build complete 9-cell regions from. Therefore, R1C4 must be a 4 (and is a complete cage cell). R2C2 must now escape down column 2 to R4C2, collecting R3/4C1 as well.

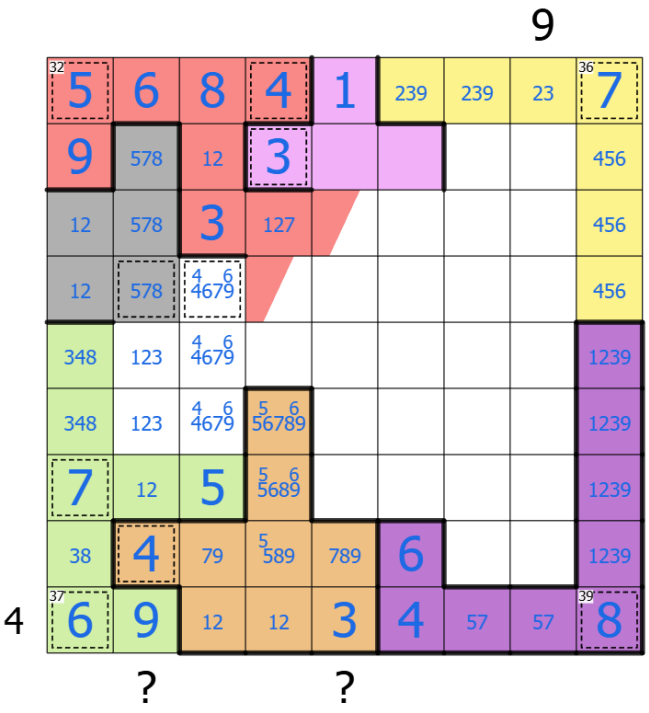
23. The red region cannot now collect R4C3 – if it did, then R4C3 must be a 7 (as it already 'sees' 4 cells), but this would make the red region at least 11 cells large. The red region therefore bends into R3C4, taking its 8th cell. R1C3 must not be a 6 (as it doesn't contain a cage), sorting the 6-8 pair in row 1.

24. The 9th red cell is either in R3C5 or R4C4. This means that neither R2C3 nor R3C4 can be a 3 (as they will both 'see' 3 cells and would require a cage).

25. R2C4 must be a new region and needs to escape into R2C5, therefore picking up R1C5 in the process. R2C4 is also a caged cell, meaning that it can only have the value of 2 or 3 (as 4, 5 and 6 are already accounted for in column 4). The important observation here is that whatever digit goes in R2C4 must then be placed in R3C3. If R2C4 is a 2, then its region must bend down into R3C5 – cutting off the red region in the process and making R3C3 the only red cell not seen by R2C4. Similarly, if R2C4 was a 3 then the only red cell that could contain a 3 (noting the 3 in R9C6) would be R3C3.



26. R2C4 must therefore be a 3 – if it were a 2, then there would be a 2 placed in R2C4 and R3C3, which would break the 1-2 pair in row 9. This leaves the 3 unfilled red cells as a 1-2-7 triple. If the red region took R3C5, then R2C3 must be a 1 or 2 (due to R3C1). On the other hand, if the red region took R4C4 then R2C3 must also be a 1 or 2 (due to R9C4). Therefore, R2C3 is always a 1 or 2 and R7C3 must be a 5. The rest of column 3 can now be pencil-marked.



27. The 9th green cell now must be the 1/2 in R5/6C2, creating a region border between the two cells.

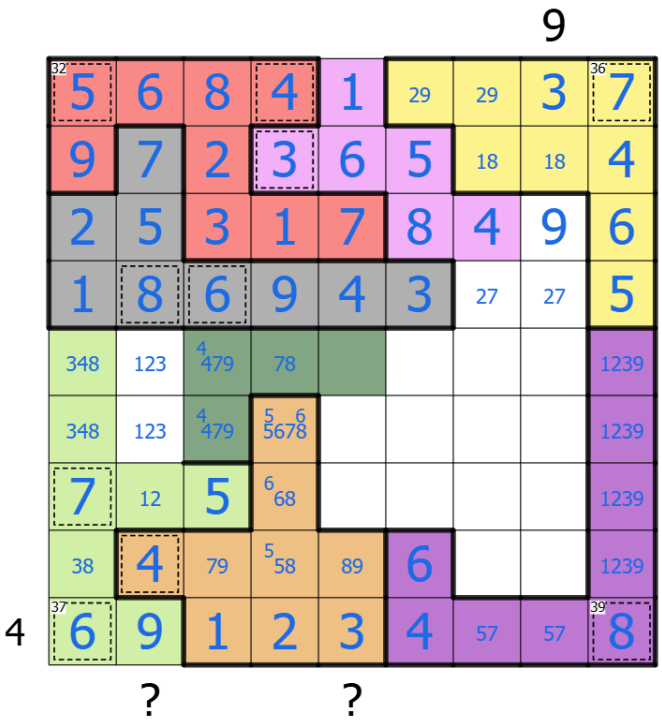
28. R4C3 cannot be a 9, as it doesn't 'see' 9 available cells. If R4C3 was a 7, then it would block off the grey region and prevent it from reaching its required quota of 9 cells. So R4C3 is a 4 or 6.

29. How does the grey region not take R4C3? It would have to duck down into R5C2 and across to R5C3, thereby taking R6C3 and R5C4 to reach 9 cells. Due to the pencil-marks in column 4, R5C4 must be a 9 – making R5/6C3 a 4-6 pair and breaking R4C3.

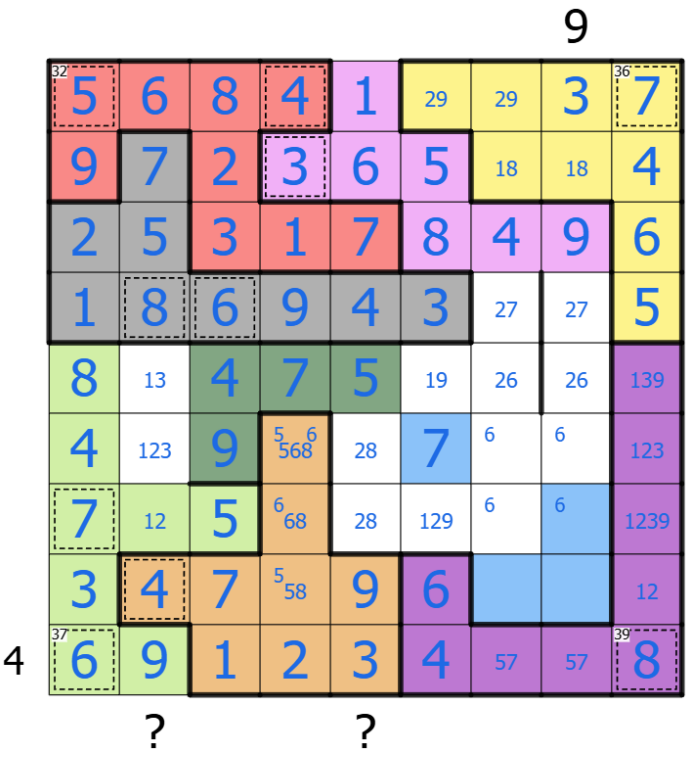
30. So R4C3 is grey. Now, how does the grey region take R5C3? If it did, then it must also take R6C3 and one of R5/6C2. But now R4C3 'sees' 5 cells, not 4 or 6. This means that the grey region cannot take R5C3 and must take R4C4 (thereby finishing the red region) and R5/6C3 must belong to a new region, which must escape into the open grid area through R5C4. This then forces the grey region into R4C5, meaning R4C3 now 'sees' at least 5 cells so must be a 6 – completing the grey region and allowing sudoku to fill in a lot of the grid.

31. The digits in the grey region can be filled in (apart from the 1-2 pair in column 1). The pink region can now be extended into row 3. Now, if R2C7/8 were not part of the yellow region then they would block off yellow in their attempt to reach 9 cells. They must therefore be the final 2 yellow cells and be a 1-8 pair.

32. R1C8 must now be a 3 (for the Numbered Rooms clue) and the 1-8 pair in yellow allows for a lot of the 1-2 pairs in the grid to be disambiguated.



33. If the pink region did not take R3C8, then to avoid R3C7 'seeing' 4 pink cells it must take R4/5/6C7. However, pink is missing a 2-7-9 triple which would (in this scenario) break R1C7 – so pink must take R3C8. Now R3C7 'sees' 3 cells, meaning it cannot take both of R4C7/8. It must therefore either take R4/5C7 or R4/5C8.



34. Where is 7 in row 8? There is a 7 x-wing in rows 4 and 9, meaning the only available cell is R8C3. This places a 9 in R8C5 and a 7 in R5C4. Now, the pink region must collect its missing 7 from row 4 and its missing 2 from row 5 (either R5C7 or R5C8).

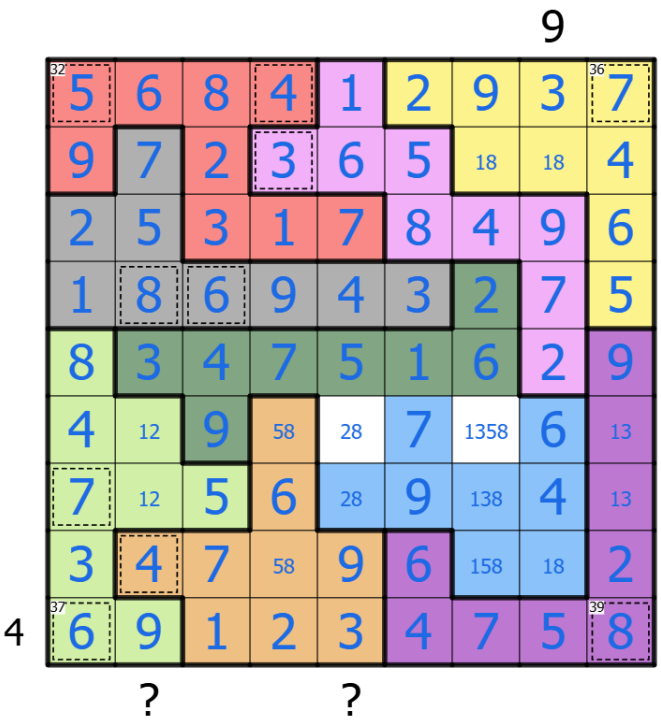
35. Where is 6 in row 5? It can only go in R5C7/8, where it must accompany the pink 2. This then places 5 in R5C5. Pencil-marking column 6 leads to a 7 in R6C6 and a 1-3-4-9 quadruple in row 5, completing columns 1 and 3.

36. So far, 8 regions have been discovered in the puzzle. This means that the 7 in R6C6 must be a part of the 9th region (as all other regions have a 7 allocated to them). The bottom-right corner of the unfilled region must also be part of this final region, as none of the existing regions can get to these cells.

37. The dark green region needs to reach a 6, but it only has 4 cells available (remembering that one of R5/6C2 must be dark green). Therefore, the only way it does not pass over the blue 7 is if it can get a 6 from R7C7. However, in doing this it must pass through R7C6 – which would then be a 1 or 2 (creating a 1-2 pair in row 7), as dark green already contains a 9. But this means that R7C7 would be 6, R7C4 would be an 8 and R7C5 could not be filled.

38. Dark green must therefore pass over the blue 7 into R5C7 – which then forces the pink region down into column 8. This allows the 2-6 and 2-7 pairs to be disambiguated, allowing most of the grid to be completed by sudoku.

39. The dark green region contains 8 cells, meaning that most of the remaining grid can be coloured blue. If R6C7 was blue, then the 2 in R4C7 would require a cage (as it would 'see' two dark green cells). This completes the region disambiguation, and the rest of the puzzle can be completed by sudoku.



The final solution to 'Say What You See' is pictured below: