Rank X-Sums – Full Solution Guide

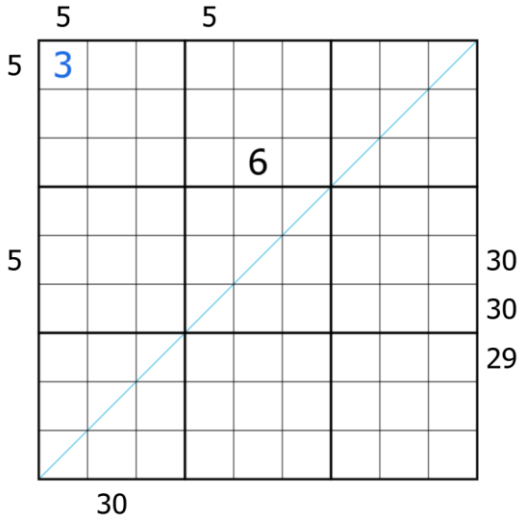
This is a full solution guide to my puzzle *Rank X-Sums*, and so spoilers are ahead.

Rules

- Normal sudoku rules apply: Place the digits 1 to 9 once each in every row, column, and 3x3 box
- **Diagonal**: The positive diagonal, marked with a blue line, contains a set of digits 1 to 9
- **Rank X-Sums**: Each row or column has an X-Sum value, which is the sum of the first X digits in that row or column, where X is the digit in the first cell. Clues outside the grid give the rank of those values from 1 (lowest) to 36 (highest). Two rows/columns with the same X-Sum value will share the lowest rank
    - (For example if the lowest X-Sum values were in order: 1, 4, 4, 4, 7, 7, 9 then these would have the clues 1, 2, 2, 2, 5, 5, 7 respectively)

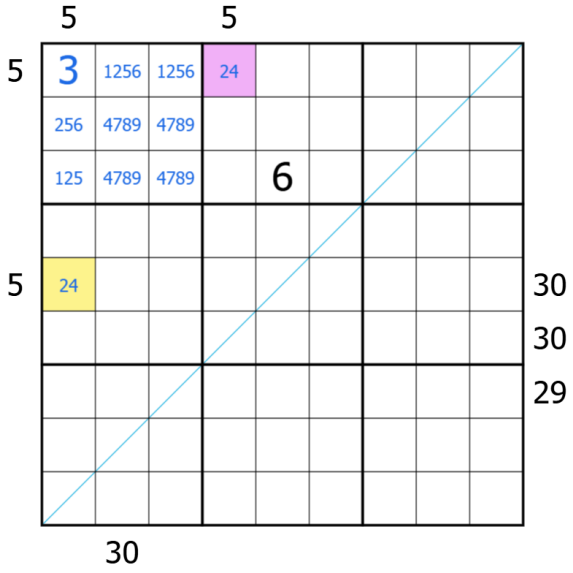
Solve Path

- Meta: There are 36 rows/columns in a sudoku
- 4 of these will start with a 1, and these have an X-Sum value of 1
- 4 of these will start with a 9, and these have an X-Sum value of 45
- Hence ranks 1 to 4 are all 1s, and ranks 33 to 36 are all 45s
- There are four X-Sums that begin with a 2. The maximum these can be is 11 with a 2-9
- The rank 5 clues need to be all the same X-Sum value, and these are the lowest rank in the puzzle that isn't a 1
- R1C1 can't be 2, as the second digit would be placed twice in box 1
- R1C1 can't be 4, as the only way to make a total no more than 11 in 4 cells using a 4 is 1234, but this would use five digits from 1234 in box 1
- Hence R1C1 is a 3, and there are two different ways to add up to our total

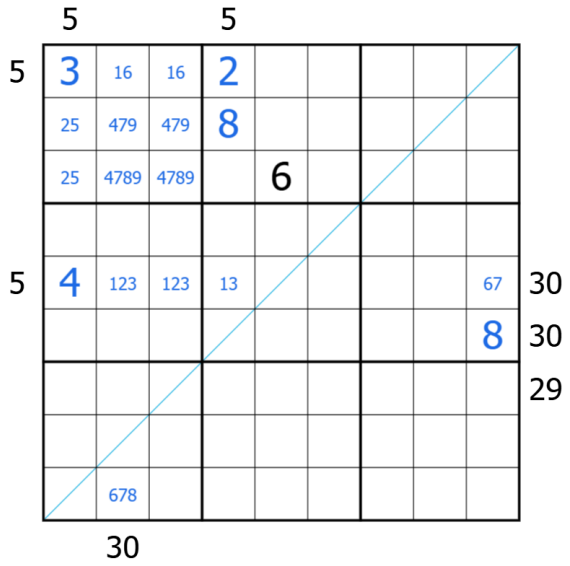


- R1C4 and R5C1 are also rank 5. These are from 2 and 4

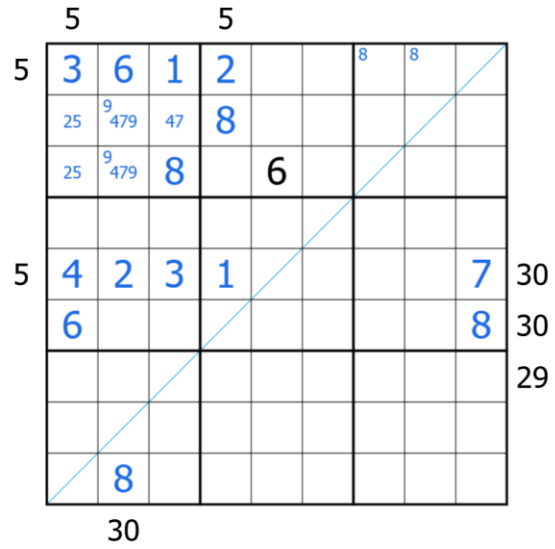
- These can't both be 2, as otherwise one of the X-Sums in box 1 is minimum 345 which sums to 12 which is too high
- These can't both be 4 as this would place 5 digits from 1234 in column 5
- So these are a 24 pair
- Now our lowest rank is known, rank 5 is a value of 10 as this must be made from sums starting with 2, 3 and 4
- Meta: Every X-Sum starting with a 2 must be followed by either 8 or 9



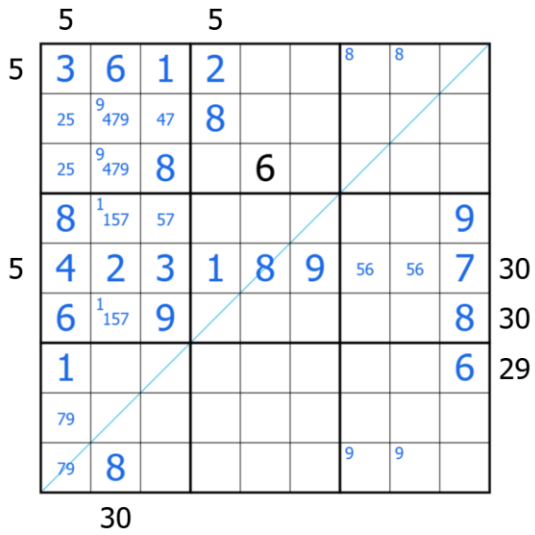
- Meta: We have three 30 clues. These are the highest ranks other than those which start with a 9
- There are X-Sums which begin with 8, and these have minimum value 36, so the rank 30 must have value strictly greater than 36
- Rank 29 has value strictly less than rank 30, but due to the ties rule, it also is strictly greater than rank 28, and is the only row/column to have this X-Sum value
- Can R5C9 and R6C9 not contain 8? If these were a 67 pair the outies in box 4 would sum to at least 15, so the maximum possible values are 37, but we would have rank < 29 which started with 8 so it would be impossible to fulfil the rank 29 clue
- Hence one of R5C9 and R6C9 is 8. Can it be R5? By outie math the x-sum value would be 41 or 43, but it is impossible to fulfil the C2 clue as either of these values (easier to see), and impossible to fulfil the R6 clue (harder to see but still true)
- Hence R6C9 is 8, and R5C9 is from 67
- Now R5C1 can't be 2. If it were, it would be followed by 8, and the rank 30 X-Sum would be value 35 which is impossible
- So R5C1 is 4



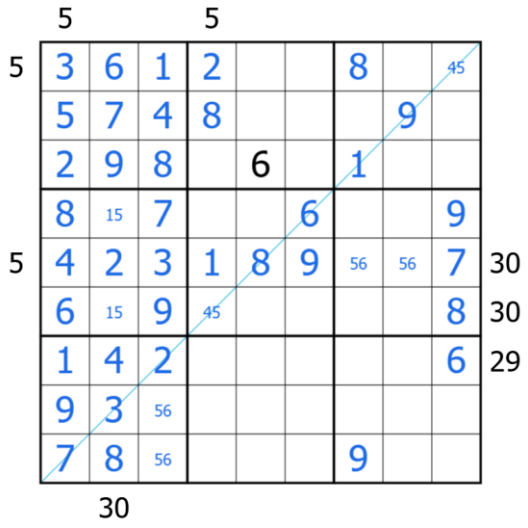
- By outie math, R6C1 = R1C2 (or R1C1 + R2C2, or R1C2 + R2C2 + R3C2 depending on the value of R9C2)
- The only option that works is R1C2 = R6C1 = 6, and R9C2 = 8
- Now by math R5C9 is 7, and R5C2 is 2



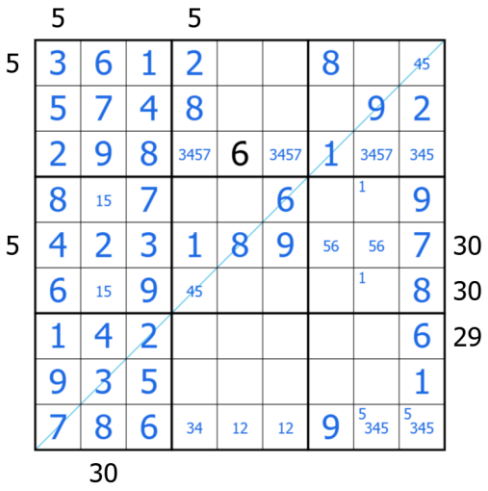
- There are two remaining X-Sums starting with 8, and these have value of no more than 37. Hence these have value 36, and the 8 is opposite a 9
- 8 is placed in box 4, and 9 opposite
- 8 is placed on the diagonal, then 9 in row 5
- Rank 29 is higher than these X-Sums, so it has value of 37 or 38, hence R7C9 is a 6, and R7C1 is a 1 as other low digits are used up in the column



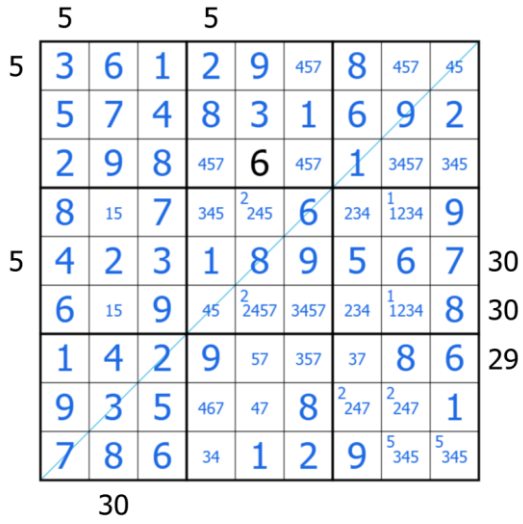
- Due to 8s in row 1, we have known 9s in row 9
- This places 7 in R9C1
- The column 1 X-Sum can't exceed 37 due to our known ranks, so R2C1 is 5, which places 9 due to our known low X-Sums
- As C1 has an X-Sum value of 37, rank 29 must be strictly greater and is thus 38 placing 2 and 4 in box 7
- 1, 9 and 6 are placed on the diagonal
- This forces 8 and 9 into column 7



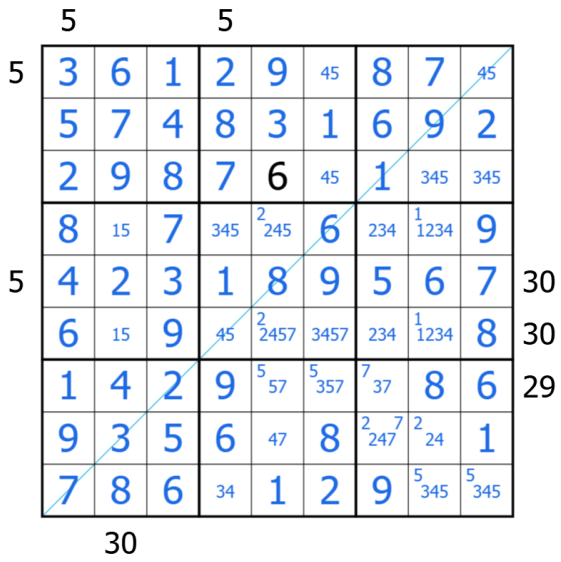
- Now consider the 7 X-Sum in row 9. The outies need to sum to at least 8, and see 6789, so these are a 35 or 45 pair
- Places 6 in R9C3, and leaves a 12 pair in R9
- Places 1 in C9, and creates a triple placing 2 in C9, which is followed by a 9 validating our X-Sum



- 2 in R9 must be followed by either 8 or 9. By sudoku this is followed by 8 and only works with one orientation
- Places 1 in box 2
- Remaining 8s and 9s can be placed
- R2C5 is a naked 3 and various sudoku steps follow



- Quad in C4 places 6 in box 8
- Then 7 placed in C4, and then in box 3



- Now consider the C8 clue, to avoid the X-sum being too large, R8C8 and R8C9 must sum to at least 8
- Alternatively, to avoid R3 being too low, R3C9 is not 3, otherwise this would sum to 8 or 9